Booster, AGS, and RHIC Parameters for the 2004–2005 RHIC Run

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The Tables in this note contain the nominal parameter values for the 2004–2005 RHIC Run. Copper ions are to accelerated and collided in RHIC.

1 Basic Parameters

A Copper ion with charge eQ has $N = 63$ Nucleons, $Z = 29$ Protons, and $(Z - Q)$ electrons. (Here Q is an integer and e is the charge of a single proton.) The mass and energy are

$$m = au - Qm_e + E_b/c^2, \quad E = \sqrt{p^2c^2 + m^2c^4} \tag{1}$$

where $a = 62.9296011$ is the atomic mass [1, 2] of the neutral Copper atom, $u = 931.494013$ MeV$/c^2$ is the unified atomic mass unit [3], $m_ec^2 = .510998902$ MeV is the electron mass [3], and p is the momentum. E_b is the binding energy of the Q electrons removed from the neutral Copper atom. This amounts to 1.306340 keV for the Cu^{11+} ion and 45.038425 keV for the fully stripped ion [4]. The kinetic energy is defined to be

$$W = E - mc^2. \tag{2}$$

In terms of W, the momentum and energy are

$$cp = \sqrt{W^2 + 2mc^2W}, \quad E = mc^2 + W. \tag{3}$$

The magnetic rigidity of the ion in units of Tm is

$$B\rho = kp/Q \tag{4}$$

where $k = 10^9/299792458$ and p is the momentum in units of GeV/c. The relativistic parameters β and γ, and the revolution frequency of the ion are

$$\beta = cp/E, \quad \gamma = E/(mc^2), \quad f = c\beta/(2\pi R). \tag{5}$$

Here R is the machine radius, defined to be the closed orbit circumference divided by 2π. The angular frequency is $\omega = 2\pi f$. We also define the phase-slip factor

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2} \tag{6}$$

where γ_t is the transition gamma.

2 RF Parameters

The stationary bucket area is

$$A_S = 8\,\frac{R_s}{hc}\left\{\frac{2eQV_gE_s}{\pi h|\eta_s|}\right\}^{1/2} \tag{7}$$

where h is the RF harmonic number, V_g is the total RF gap voltage per turn, and the subscript "s" denotes parameter values for the synchronous particle.

The half-height of a bucket is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S\,|(\pi - 2\phi_s)\sin\phi_s - 2\cos\phi_s|^{1/2} \tag{8}$$

where ϕ_s is the synchronous phase.

The synchronous phase is given by

$$V_g \sin\phi_s = 2\pi R_s \rho_s \dot{B}/c \tag{9}$$

where ρ_s is the radius of curvature, B is the magnetic field and $\dot{B} = dB/dt$. Employing Gaussian units (R_s and ρ_s in cm, $c = 2.99792458 \times 10^{10}$ cm/s, and $\dot{B}$ in G/s) gives $V_g \sin\phi_s$ in Statvolts. Multiplying by 299.792458 then gives $V_g \sin\phi_s$ in Volts.

The width of a bucket is

$$\Delta t = \frac{|\pi - \phi_s - \phi_e|}{h\omega_s} \tag{10}$$

where the phase ϕ_e satisfies

$$\cos(\pi - \phi_s) - \cos\phi_e = -(\pi - \phi_s - \phi_e)\sin\phi_s. \tag{11}$$

The area of a bucket is

$$A_{\text{bk}} = \alpha(\phi_s) A_S \tag{12}$$

where

$$\alpha(\phi_s) = \frac{\sqrt{2}}{8}\int_{\phi_L}^{\phi_R} |(\pi - \phi_s - \phi)\sin\phi_s - \cos\phi_s - \cos\phi|^{1/2}\, d\phi. \tag{13}$$

Below transition we have $\phi_e < \pi - \phi_s$ and the limits of integration are $\phi_L = \phi_e$ and $\phi_R = \pi - \phi_s$. Above transition we have $\pi - \phi_s < \phi_e$ and the limits of integration are $\phi_L = \pi - \phi_s$ and $\phi_R = \phi_e$. The integral $\alpha(\phi_s)$ must be evaluated numerically. An approximate expression is [5]

$$\alpha(\phi_s) \approx \frac{1 - \sin\phi_s}{1 + \sin\phi_s}. \tag{14}$$

The synchrotron frequency for small-amplitude oscillations about ϕ_s is

$$F_s = \frac{c}{2\pi R_s}\left\{\frac{-h\eta_s eQV_g\cos\phi_s}{2\pi E_s}\right\}^{1/2} \tag{15}$$

and the corresponding synchrotron tune is $Q_s = 2\pi F_s/\omega_s$.

Let ϕ_l and ϕ_r be the phases at the left and right boundries of a bunch matched to a bucket. We have

$$\phi_l < \phi_s < \phi_r \tag{16}$$

and the width of the bunch is

$$\Delta t = \frac{\Delta\phi}{h\omega_s}, \quad \Delta\phi = \phi_r - \phi_l. \tag{17}$$

In terms of $\Delta\phi$ and ϕ_s we have

$$\phi_r = \frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\} \tag{18}$$

and

$$\phi_l = -\frac{\Delta\phi}{2} + \arcsin\left\{\frac{\Delta\phi\sin\phi_s}{2\sin(\Delta\phi/2)}\right\}. \tag{19}$$

Note that if $\Delta\phi$ is small we have

$$\sin(\Delta\phi/2) \approx \frac{\Delta\phi}{2}, \quad \frac{\Delta\phi \sin\phi_s}{2\sin(\Delta\phi/2)} \approx \sin\phi_s \tag{20}$$

and

$$\phi_l \approx \phi_s - \frac{\Delta\phi}{2}, \quad \phi_r \approx \phi_s + \frac{\Delta\phi}{2}. \tag{21}$$

The half-height of the bunch is

$$\Delta E = \left(\frac{h\omega_s}{8\sqrt{2}}\right) A_S \, |\cos\phi_r - \cos\phi_s + (\phi_r - \phi_s)\sin\phi_s|^{1/2}. \tag{22}$$

The area of the bunch is

$$A_{\rm b} = F(\phi_s, \Delta\phi)\, A_S \tag{23}$$

where

$$F(\phi_s, \Delta\phi) = \frac{\sqrt{2}}{8} \int_{\phi_l}^{\phi_r} |\cos\phi_l - \cos\phi + (\phi_l - \phi)\sin\phi_s|^{1/2} \, d\phi. \tag{24}$$

The integral $F(\phi_s, \Delta\phi)$ must be evaluated numerically. If $\Delta\phi$ is small we have

$$F(\phi_s, \Delta\phi) \approx \frac{\pi}{64} (\Delta\phi)^2 \, |\cos\phi_s|^{1/2}. \tag{25}$$

3 Lattice Parameters

Parameter	Booster	AGS	RHIC	Unit
C_I	C_b	C_a	C_r	m
C_E	$C_a/4$	$4C_r/19$	C_r	m
ρ	13.8656	85.378351	242.7806	m
γ_t	4.806	8.5	22.89	
Q_H, Q_V	4.757, 4.777	8.78, 8.72	28.19, 29.18	
Max β_H, β_V	13.5, 13.2	22.3, 22.2	48.6, 47.4	m
Max D_H	2.90	2.17	1.81	m

Here C_I and C_E are the circumferences of the closed orbits in the machines at injection and extraction respectively. C_b, C_a, and C_r are the circumferences of the "design" orbits in Booster, AGS, and RHIC respectively. These are

$$C_b = 201.780, \quad C_a = 2\pi(128.4526), \quad C_r = 3833.845181 \tag{26}$$

meters. Note that $4C_r/19 = 2\pi(128.45798)$ which gives an AGS radius at extraction approximately 5 mm larger than the "design" AGS radius (128.4526 m) reported by Bleser [6]. The other Booster and AGS parameters were obtained from MAD runs. The RHIC parameters are taken from Ref. [7] and from MAD runs by Steve Tepikian. (The maximum β_H, β_V, D_H listed for RHIC are the maxima in the arcs.)

4 Assumptions

The parameters values listed in Sections 5–7 are calculated assuming that:

1. The magnetic rigidity of the Cu^{11+} ions at Booster injection is $B\rho = 1.300433$ Tm.

2. The frequency at Booster extraction is $hf = 3.850$ MHz at harmonic $h = 6$.

3. The frequency at Ags injection is $hf = 3.831$ MHz at harmonic $h = 24$.

4. The magnetic rigidity of the Cu^{29+} ion at RHIC injection is the same as that of a proton with γ_p such that $G\gamma_p = 46.5$. Here $G + 1 = 2.792847337(29)$ and the proton mass is $m_p = 0.938271998(38)$ GeV/c^2 as reported in Ref. [8]. Thus $\gamma_p = 25.93639684$ and the proton momentum and energy are $P_p = m_p c\sqrt{\gamma_p^2 - 1} = 24.3173002$ GeV/c and $E_p = m_p c^2 \gamma_p = 24.3353949$ GeV. The rigidity is then $B\rho = kP_p = 81.1137824$ Tm.

5. The energy of the Cu^{29+} ion at RHIC Store is 100 GeV per nucleon.

Please note that more digits are given for some parameters in Sections 5–7 than would be warranted by the precision with which the parameters could be measured; this is done for computational convenience. The notation "$/N$" in the Units column of the tables means "per nucleon".

5 Copper Parameters in Booster

Parameter	Injection	Extraction	Unit
Q	11	11	
m	58.612926983	58.612926983	GeV/c^2
W	156.67485/63	101.15420	MeV/N
cp	68.070795	445.48021	MeV/N
E	0.93285082	1.0315181	GeV/N
$B\rho$	1.300433	8.5105098	Tm
β	0.072970718	0.43186852	
$\gamma - 1$	0.0026730425	0.10872542	
η	-0.951	-0.770	
ϵ_H (95%)	13.5π	13.5π	mm mrad
ϵ_V (95%)	6.4π	6.4π	mm mrad
h	6	6	
hf	0.6504927	3.850	MHz
R	$201.780/(2\pi)$	128.4526/4	m

Here ϵ_H and ϵ_V are the normalized horizontal and vertical transverse emittances. These follow from the assumption that during multi-turn injection the horizontal and vertical acceptances in Booster are completely filled. The horizontal and vertical acceptances are 185π and 87π mm mrad (un-normalized) respectively.

Parameter	Injection	Extraction	Unit
V_g	0.5	30	kV
A_S	0.8576	7.763	eV s
Bdot	0	80	G/ms
ϕ_s	0	48.25	degrees
F_s	0.433	2.343	kHz
A_{bk}	0.8576	1.048	eV s
A_b	0.280	0.280	eV s
Δt	663	40	ns
ΔE	0.275	4.49	MeV

Parameter	Injection	Extraction	Unit
No. of Bunches	6	6	
Bunch Spacing	1537.296	259.740	ns
Ions/Bunch	10.25/6	8.25/6	10^9
Bunch Area	0.027/6	0.027/6	eV s/N

Capture of the injected beam in Booster occurs on a 6 ms porch at constant field. During this time the gap voltage is increased from 0 to 0.5 kV. The bunch area at Booster extraction was obtained from measurements of the bunch width on the AGS injection porch [9]. The gap voltage in Booster was determined from measurements of the synchrotron frequency [10].

6 Copper Parameters in AGS

Parameter	Injection	Transition	Extraction	Unit
Q	29	29	29	
m	58.603772735	58.603772735	58.603772735	GeV/c^2
W	0.099976301	6.9766396	10.302044	GeV/N
cp	0.44271310	7.8519488	11.193678	GeV/N
E	1.03019492	7.9068582	11.232263	GeV/N
$B\rho$	3.2080729	56.898302	81.1137824	Tm
β	0.42973722	0.99305547	0.99656480	
γ	1.1074761	8.5000	12.07486366	
η	-0.801	0.0	0.00698	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	24	12	12	
hf	3.831	4.4264207	4.4418770	MHz
R	128.4526	128.4526	128.45798	m

Here, as in the past, four Booster loads of six bunches are injected into AGS each AGS cycle. These are injected at constant field into $h = 24$ stationary buckets. The beam is then debunched adiabatically and rebunched at harmonic 4. Acceleration to top energy occurs on harmonic 12. The bunches are extracted on flat-top at constant field.

The bunch areas at AGS injection and extraction were obtained from measurements of the bunch width [9]. The large increase in longitudinal emittance between Booster and AGS is due to the deliberate mismatch of the AGS RF buckets to the incoming bunches on the injection porch. This mismatch significantly reduces the severe drool loss seen on the porch when the buckets are matched to the bunches. Gap voltages as high as 220 kV are required to match the buckets to the incoming bunches. For an incoming bunch width of 40 ns this gives a matched bunch area of 0.752

eV-s. Comparing with the bunch area of 0.280 eV-s at Booster extraction we conclude that the increase in longitudinal emittance upon passing through the BTA stripping foil is as high as a factor of 0.752/0.280 = 2.7.

Parameter	Injection	Extraction	Unit
V_g	19	144	kV
A_S	4.914	1354	eV s
Bdot	0	0	G/ms
ϕ_s	0	180	degrees
F_s	1.894	0.104	kHz
A_{bk}	4.914	1354	eV s
A_b	1.772	6×4.16	eV s
Δt	119	22	ns
ΔE	9.71	722	MeV

Parameter	Injection	Extraction	Unit
No. of Bunches	24	4	
Bunch Spacing	261.028	675.390	ns
Ions/Bunch	7.0/6	6.75	10^9
Bunch Area	0.169/6	0.396	eV s/N

The intensities (Ions/Bunch) listed in the tables were obtained from careful measurements with the Booster and AGS circulating beam current transformers [11].

7 Copper Parameters in RHIC

Parameter	Injection	Transition	Store	Unit
Q	29	29	29	
m	58.603772735	58.603772735	58.603772735	GeV/c^2
W	10.302044	20.362485	99.069781	GeV/N
cp	11.193678	21.272375	99.995673	GeV/N
E	11.232263	21.292704	100.000000	GeV/N
$B\rho$	81.1137824	154.147977	724.607889	Tm
β	0.99656480	0.99904526	0.99995673	
γ	12.07486366	22.8900	107.501611	
η	-0.00495	0.0	0.00182	
ϵ_H (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
ϵ_V (95%)	$\leq 10\pi$	$\leq 10\pi$	$\leq 10\pi$	mm mrad
h	360	360	360	
hf	28.05396013	28.12378671	28.14944534	MHz
$2\pi R$	3833.845181	3833.845181	3833.845181	m

Parameter	Injection	Store	Unit
No. of Bunches	60	60	
Bunch Spacing	213.874	213.148	ns
Ions/Bunch	6.0	6.0	10^9
Bunch Area	0.4	0.7	eV s/N

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